

process is the same in every case, whatever may be the original form of the figures proposed; for by terminating the process of conversion in a square, the result is always obtained by the simple multiplication of two quantities, or factors, which are equal to one another.

Land-surveyors have long been in the habit of reducing their figures to a triangle; this is, indeed, a very convenient process for the purpose of calculation merely; but it is not at all adapted for a criterion or standard of comparison, because the triangle is a figure of which the magnitude is not easily assignable by the eye, nor does it lead us to that principle by which a given figure may be increased and decreased in any proposed ratio at pleasure, or even multiplied and divided geometrically, in the same manner, and with greater ease and facility than the same processes can be performed by numbers themselves.

The process of conversion, which these papers are intended to illustrate, is not limited in its application; it extends to all plane figures whatever, and in some instances even to the surfaces of solids, such as the cylinder, the sphere, the cone, the conic frustum, and to almost all the solids of revolution, as well as to those contained under regular plane boundaries. It is likewise applicable to the demonstration of a numerous class of geometrical propositions, thereby constituting a very delightful exercise.

Could practical men be therefore induced to direct their attention to the subject, it would be found materially to their advantage, not only as regards the solution of problems which offer themselves in the course of their operations, but also as a means of rendering them familiar with the chief propositions of elementary geometry, without being subjected to the tedious and irksome drudgery of tracing the demonstrations after the strict logical method in which they are usually given. It is true that the demonstrations given in this way are merely tentative, or at the most mechanical, but they are not the less convincing by being derived from measurement; and the ease with which they are obtained by the principles of conversion is a very high recommendation as regards its practical utility.

PROBLEM.—To construct a square that shall be equal in area to a given rectangular parallelogram; that is, to convert a given rectangular parallelogram into a square of equal area.

Example. Let ABCD (fig. 1) be the given rectangular parallelogram, of which the containing sides are AB and BC; it is required to convert the rectangular parallelogram ABCD into a square of equal area.

Construction. Produce the side AB directly forward to any convenient length at pleasure, or as far as may be found necessary, and on the point B as a centre, with the side BC as radius, describe the quadrant arc BmC meeting AB produced in I, and making BI equal to BC, so that AI is equal to AB and BC taken together, or equal to the sum of the sides of the parallelogram continued out in one straight line.

Bisect the sum of the sides AI in H, and on H as a centre, with HA or HI as radius, describe the semi-circle AGI meeting the production of the side CB in G; then is BG the side of the square sought. Therefore, if upon BG, thus determined, the square BGEF be constructed, it will be equal in area to the rectangular parallelogram ABCD, as required by the problem.

Demonstration. Since the straight line, AI, which is equal to the sum of the sides of the rectangular parallelogram AB and BC, is equally divided in the point H, and unequally in the point B, the rectangle under AB and BI together with the square of BH, is equal to the square of HA or HG, the radius of the semi-circle; but the squares of BH and BG are together equal to the square of HG, because GBH is a right angle; therefore, the squares of BH and BG are together equal to the rectangle under AB and BI, and the square of BI; take away from these equals the square of BH, which is common, and there remains the square of BG, which is equal to the rectangle under AB and BI, or AB and BC, because BI, by construction, is equal to BC; consequently, the square upon BG, that is, the square BGEF, is equal in area to the rectangular parallelogram ABCD. This construc-

tion may be otherwise more briefly and elegantly demonstrated, but we prefer the method here employed, as being most in accordance with the plan of our present undertaking.

PROBLEM.—To construct a square that shall be equal in area to a given rhomboid or oblique parallelogram; that is, to convert a given oblique parallelogram into a square of equal area.

Example.—Let ABCD (fig. 2) be the given rhomboid or oblique-angled parallelogram, of which the containing sides are AB, BC, CD, and DA; it is required to convert the parallelogram ABCD into a square of equal area.

Construction.—From A and B, the extremities of one of the longer sides, AB, let fall the perpendiculars AK and BL, meeting DC in K, and its extension in L, thereby forming the rectangular parallelogram ABLK, equal to the given rhomboid ABCD. Produce AB directly forward to any indefinite extent, and on B as a centre, with BL as a radius, describe the quadrant arc LmI, meeting AB produced in I, and making BI equal to BL, the side of the derived rectangular parallelogram, so that AI shall be equal to the sum of its containing sides extended in the same straight line.

Bisect AI in H, and on H as a centre, with HA or HI as a radius, describe the semi-circle AGI meeting the side LB produced in G; then is BG the side of the required square. Upon BG, as thus determined, construct the square BGEF, and it will be equal in area to the given rhomboid ABCD.

Demonstration.—Since the rhomboid ABCD and the rectangular parallelogram ABLK are placed upon the same base, AB, and lie between the same parallels, AB and DL, they are equal to one another, the right-angled triangle ADK, which is cut off from the rhomboid by the perpendicular AK, being equal to the triangle BCL, which is applied to it, by CL, the extension of the side DC, and the perpendicular BL meeting that extension. Again, since the straight line AI, which is equal to the sum of the sides AB, BL, of the rectangular parallelogram, ABLK, is divided equally in the point H and unequally in B, the rectangle or product under the two unequal parts AB, BI, together with the square of BH, the part between the points of section, is equal to the square of HI or HG; but the square of HG is equal to the sum of the squares of BH and BG, since HBG is a right-angled triangle, of which HG is the hypotenuse; consequently, the rectangle, or product, under the parts AB and BI, together with the square of BH, is equal to the sum of the squares of BH and BG; take away the square of BH from these equals, and there remains the rectangle or product, under the parts AB and BI, equal to the square of BG; but BI, by the construction, is equal to BL, the side of the rectangular parallelogram ABLK; therefore, the square of BG, that is, the square BGEF, is equal in area to the rectangle ABLK, and, consequently, to the rhomboid ABCD.

PROBLEM.—To constitute a square that shall be equal in area to a given rhombus, or an equilateral four-sided figure; that is, to convert a given rhombus or equilateral four-sided figure into a square of equal area.

Example.—Let ABCD (fig. 3) be the given rhombus or equilateral figure, of which the containing sides are AB, BC, CD, and DA; it is required to convert the rhombus ABCD into a square containing the same or an equal area.

Construction.—From A and B, the extremities of the upper side of the rhombus AB, let fall the perpendiculars AK and BL, meeting the opposite or lower side DC in K, and its extension in L, thereby forming the rectangular parallelogram ABLK, equal in area to the given rhombus ABCD. This is obvious, for parallelograms upon the same base and between the same parallels are equal; but the rectangle ABLK and rhombus ABCD are upon the same base AB, and between the same parallels AB and DL; therefore they are equal.

Upon A as a centre, with AK as a radius, describe the circular quadrant KmI, meeting BA produced in I, and making AI equal to AK, the side of the rectangle ABLK, so that BI shall be equal to the sum of its containing sides extended out in the same straight line. Bisect BI in H, and on H as a centre, with HB or HI as a radius, describe the semi-circle BGI, meeting KA produced in G; then is AG the side of a square containing an area

equal to the rectangle ABLK, and consequently equal to the given rhombus ABCD. On AG describe the square AGEF, and it will be equal to the rectangle ABLK.

For draw GB and GI; then because BGI is an angle in a semi-circle, it is a right angle, and since GA is perpendicular to the diameter IB, the triangles IGA and GBA are similar, and consequently, their corresponding sides are proportional, so that IA is to AG, as AG is to AB; that is, the rectangle under IA and AB is equal to the square of AG; but IA is equal to AK; therefore, the rectangle under AK and AB is equal to the square of AG; hence the square AGEF is equal to the rectangle ABLK, and consequently, equal to the given rhombus ABCD.

PROBLEM.—To constitute a square that shall be equal in area to a given right-lined triangle; that is, to convert a given right-lined triangle into a square of equal area.

Note.—The solution of this problem branches itself into three cases, for the triangle may be either acute-angled, obtuse-angled, or right-angled; but in all the cases, the principles of solution are the same, excepting only the simplicity of operation.

Example.—Let ABC (fig. 4) be the given right-lined triangle of which the sides are AB, BC, and AC; it is required to convert the triangle ABC into a square of equal area.

Case 1st.—When the triangle is acute-angled.

Construction.—Through C, the vertex of the given triangle ABC, draw the indefinite straight line CI parallel to the base AB, extending it both ways if found necessary so to do, in consequence of the form of the triangle. Bisect the base AB in L, and at the point of bisection L, and also at B, the extremity of the base, erect the perpendiculars LK and BD, meeting the indefinite parallel CI in the points K and D, and constituting thereon the rectangular parallelogram BDKL equal to the given triangle ABC. On the point D as a centre, with DB as a radius, describe the quadrant arc BmI, meeting the straight line CI in the point I, and making DI equal to DB, the side of the equivalent rectangle; then is KI the sum of the two sides KD and DB continued in one straight line. Bisect KI in H, and on H as a centre, with HI or HK as a radius, describe the semi-circle KGI intersecting BD, the side of the rectangle produced in G; then is DG the side of a square of equal area with the rectangle BDKL, and also the triangle ABC. Upon DG, as thus determined, construct the square GEFD and the thing required by the problem is accomplished.

Demonstration.—Since the base AB of the triangle ABC is bisected in L, and because the straight lines AB and CD are parallel to one another, the rectangular parallelogram BDKL, which is constituted on half the base BL, is equal to the triangle ABC. Again, since the straight line IK is equally divided in the point H and unequally in D, the rectangle or product under ID and KD, together with the square of DH, is equal to the square of HI or HG; but the square of HG is equal to the sum of the squares of DH and DG, because the angle GDH is a right-angle; consequently, the rectangle or product under ID and KD, together with the square of DH, is equal to the sum of the squares of DH and DG; from these equals take away the square of DH which is common, and there remains the rectangle under ID and KD equal to the square of DG; but DI by construction is equal to DB, the side of the parallelogram; therefore, the square of DG, that is, the square GEFD, is equal to the parallelogram BDKL, and consequently to the triangle ABC.

Case 2nd.—When the triangle is obtuse-angled, as ABC (fig. 5), the sides being AB, AC, and BC, respectively, it is required to convert the triangle ABC into a square of equal area.

Construction.—Through C, the vertex of the triangle, draw the straight line CK parallel to the base of the triangle ABC, extending it as far as necessary to accommodate the construction. Bisect the base AB in L, and through L and B, the middle point and the extremity of the base AB, draw the straight lines LD and BK respectively perpendicular to the base AB, and limited by the parallel line CK passing through C; then is LDKB a rectangular parallelogram equal to the triangle ABC.

On the point D as a centre, with DL as a